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SOLUTION OF A TWO-DIMENSIONAL CONJUGATE PROBLEM OF STABILIZED HEAT TRANSFER IN THE LAMINAR FLOW OF A LIQUID IN A CHANNEL

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A two-dimensional problem of conjugate heat exchange is solved by the method of integral boundary-value equations. Heat exchange in a body with cylindrical channels is studied.

Theoretical investigations of convective heat exchange between a solid and a liquid are generally conducted by assigning third-order boundary conditions on the solid–liquid interface. These conditions include the heat-transfer coefficient α, determined a priori. Such a formulation of the problem does not consider the mutual thermal effect of the body and liquid, and heat exchange is independent of the properties of the body or its thermophysical characteristics, dimensions, etc. Thus, it is necessary to examine a so-called conjugate problem, i.e., to simultaneously solve the equations of heat conduction in the body and liquid under the condition of equality of the temperatures and heat fluxes at the interface [1, 2].

One of the approaches to solving conjugate problems is based on the method of integral boundary-value equations [3].

Let the flat wall of the heat exchanger receive a heat flow of intensity q. The heat is removed by a liquid flowing in cylindrical channels of the same radius lying in a plane parallel to the wall. We will assume that the motion of the liquid is laminar and that the heat exchange between the liquid and solid is steady.

Using these assumptions and symmetry conditions, let us formulate the problem of determining the temperature field in the following manner:

In a two-dimensional region D (Fig. 1), consisting of two subregions D_1 (solid) and D_2 (liquid), it is necessary to find the solution to the system of equations

$$\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} = 0 \quad (x,\ y) \in D_1, \tag{1}$$

$$c_p \rho W \frac{\partial T_2}{\partial z} = \lambda_2 \left(\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} \right) \quad (x,\ y) \in D_2 \tag{2}$$

with the following boundary conditions:

$$\lambda_1 \frac{\partial T_1}{\partial n} = q \quad (x,\ y) \in CD,\ T_1 = T_2,\ \lambda_1 \frac{\partial T_1}{\partial n} = \lambda_2 \frac{\partial T_2}{\partial n} \quad (x,\ y) \in AB.$$

We have assigned $\partial T/\partial n = 0$ on the rest of the boundary. Since only the heat flux is assigned on the boundary, the temperature is determined to within a constant, chosen so that the temperature integral over the interface AB is equal to zero.

On the section of thermal stabilization, the derivative $\partial T_2/\partial z$ will be constant. From the condition of heat balance, we find $c_p \rho \frac{\partial T_2}{\partial z} = \frac{2qa}{\pi R^2 \overline{W}}$.

The velocity field for laminar flow in a channel of circular cross section is given by Poiseuille's formula [1]:

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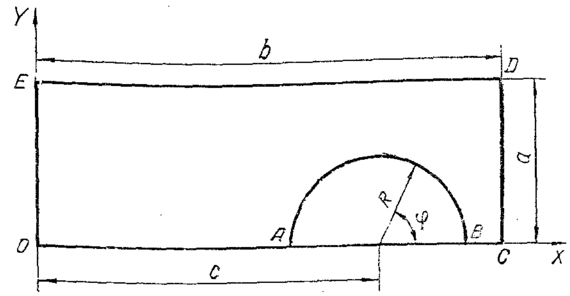

Fig. 1. Region of solution of conjugate problem (rectangle OEDC is the region D_1, while the hemisphere with the diameter AB is the region D_2).

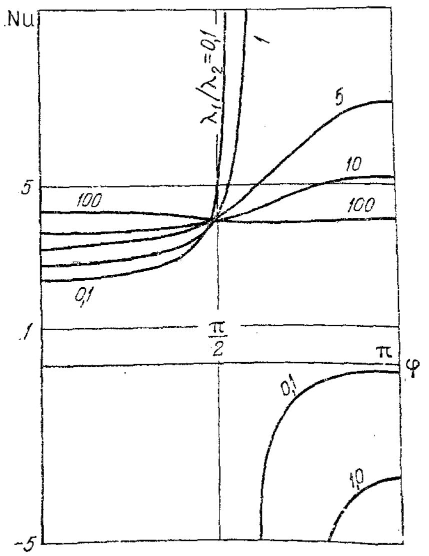

Fig. 2

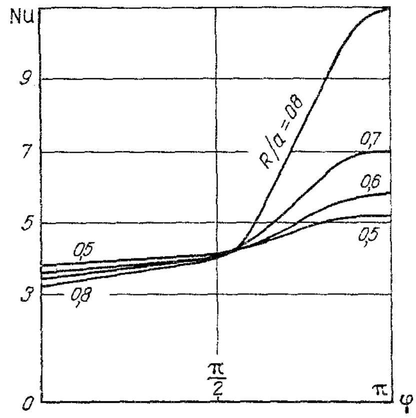

Fig. 3

Fig. 2. Dependence of the number Nu on the angle φ at the interface for different ratios of thermal conductivities.

Fig. 3. Dependence of the Nusselt number Nu on the angle φ at the interface for different channel radii.

$$\frac{W}{\overline{W}} = 2\left(1 - \frac{r^2}{R^2}\right), \quad r^2 = (x - c)^2 + y^2.$$

Allowing for these relations, Eq. (2) for the temperature of the liquid takes the form

$$\lambda_2\left(\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2}\right) = \frac{4qa}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right). \tag{3}$$

Thus, the conjugate heat-exchange problem being examined reduces to a type IV boundary-value problem for the Poisson equation in the two-dimensional composite region D. An algorithm was proposed in [4] for solving such problems based on the method of integral boundary-value equations. To use this approach, it is necessary to know the particular solution of Eq. (3). This is easily found in a polar coordinate system with its origin at the center of a circle $U = \frac{qa}{\lambda_2 \pi R^2} r^2 \left(1 - \frac{r^2}{4R^2}\right)$. Thus, the problem can be solved by the method proposed in [4].

To find the temperature field in the region D, we wrote a program in FORTRAN for the EC computer. The calculations were performed for the following dimensions: b/a = 10.0, c/a = 8.5, R/a = 0.5.

Figure 2 shows the change in the local Nusselt number: $\mathrm{Nu} = \lambda_2 \frac{\partial T_2}{\partial n} 2R \Big/ (T_2 - \bar{T})$ at the interface as a function of the polar angle φ for different values of the ratio of thermal conductivities. It has been shown in works dealing with conjugate heat exchange that the usual condition of heat exchange between the body and liquid in the form of Newton's law becomes inapplicable for certain parameters of the problem in question. A similar conclusion follows from the results shown in Fig. 2. At high values of λ_1/λ_2, the Nusselt number is nearly constant along the boundary and approaches a value of 4.36. This corresponds to the solution with a constant heat flux on the surface of a round tube [6].

With a decrease in λ_1/λ_2 to values of the order of 1.0, the Nusselt number becomes variable along the interface, and a region with negative values of the heat-transfer coefficient appears, i.e., Newton's law ceases to describe the heat exchange process.

Figure 3 shows curves depicting the dependence of the Nusselt number on the angle φ at $\lambda_1/\lambda_2 = 10.0$ for different values of the channel radius. An increase in the radius is accompanied by an increase in the nonuniformity of heat transfer about the channel perimeter, i.e., the possibility of using third-order boundary conditions, other conditions being equal, also depends on the geometry of the heat exchanger.

In conclusion, we should note that the method of integral boundary-value equations permits the solution of problems of stabilized heat exchange in bodies consisting of parts with different thermal conductivities in the presence of certain circular channels of a different form with a known velocity field.

NOTATION

x, y, z, Cartesian coordinates; R, channel radius; λ_1, λ_2, thermal conductivities of the body and liquid; T_1, T_2, temperature of the body and liquid; c_p, heat capacity of the liquid; ρ, density; W, velocity of liquid along the z axis; $\bar{W}$, mean velocity of liquid; $\bar{T}$, mean-balance temperature; q, heat flux; Nu, Nusselt number.

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